

# Non-equilibrium thermodynamics of dark energy on the power-law entropy corrected apparent horizon

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**Abstract:** We investigate the Friedmann-Robertson-Walker (FRW) universe (containing dark energy) as a non-equilibrium (irreversible) thermodynamical system by considering the power-law correction to the horizon entropy. By taking power-law entropy area law which appear in dealing with the entanglement of quantum fields in and out the horizon, we determine the power-law entropy corrected apparent horizon of the FRW universe.

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## I. INTRODUCTION

It is quite well known that black hole behaves like a black body, emitting thermal radiations, with a temperature proportional to its surface gravity at the black hole horizon and with an entropy which proportional to its horizon area [1, 2]. Further it is also known that the Hawking temperature and horizon entropy together with the black hole mass obey the first law of thermodynamics  $dM = TdS$  [3]. In the literature, the thermodynamical features of black holes have been studied by taking the dark energy into account [4–7] which is equally well defined in this scenario for the apparent horizon of the black hole, while the discussion remains no more possible for event horizon.

It is generally believed that the current universe is in accelerated expansion phase due to the presence of dark energy. The holographic dark energy (HDE) has been studied extensively in the literature [8–10] and proposed as an interesting candidate of dark energy which is motivated from the holographic principle [11]. In the derivation of HDE density  $\rho_D = 3c^2 M_p^2 L^{-2}$ , the black hole entropy  $S_{BH} = \frac{A}{4G}$ , where  $A \propto L^2$  and  $A$  represents the area of the horizon [12] plays a key role. Here  $M_p^2 = (8\pi G)^{-1}$  is the modified Planck mass and  $3c^2$  is constant and introduced for convenience.

However, this definition of entropy-area relation can be modified due to power-law corrections to entropy which arise in studying the entanglement of quantum fields in and out the horizon of the black hole [13]. The power-law corrected entropy relation is given by the equation [14]

$$S = \frac{A}{4G}[1 - K_\alpha A^{1-\alpha/2}], \quad (1)$$

where  $\alpha$  is a dimensionless constant whose value is still matter of debate, and

$$K_\alpha = \frac{\alpha(4\pi)^{\alpha/2-1}}{(4-\alpha)r_c^{2-\alpha}}, \quad (2)$$

where  $r_c$  is the crossover scale. In (1), the second term is defined to be the power-law correction to the area law mainly appear due to entanglement, when the wave function is defined in terms of ground state and excited state [13]. So the entanglement entropy of the ground state fulfils the Hawking area law while the excited state is related with the correction, and more excitations produce more deviation from the area law. For more detail see [15]. It provides strong basis to entanglement that it can be used as a source of black hole entropy.

It is important to mention that the correction term falls off rapidly with  $A$  and hence in the semi-classical limit (for large  $A$ ), the area law is recovered while in the case of small black holes the correction term will play a significant role. For instance, at low energies, in case of large horizon area, it is difficult to excite the modes and therefore the ground state modes contribute to most of the entanglement entropy. On the other hand, for small area, a large number of field modes will be excited and play a significant role in the correction causing large deviation from the area law.

In the cosmological framework, the study of non-equilibrium thermodynamics of dark energy has some interesting insights. To explore the hidden features of correction term, Das et al [13] computed leading-order corrections to the entropy of a thermodynamical system caused by small statistical fluctuations around equilibrium. They showed that one can obtain a general logarithmic correction related to the black hole entropy. The thermodynamical explanation of the interaction between HDE and dark matter is given in detail in [16] and taking the logarithmic correction to the equilibrium entropy, they derived expression for the interaction term that was consistent with the observational data. Pavon and Wang [17] proposed a system consists of dark energy and dark matter (with equilibrium entropies). They argued that if there is transfer of energy between dark energy and dark matter, the entropy of dark matter directly linked with the entropy of dark energy. Further, Zhou et al [18] studied the natural interaction between dark matter and dark energy in the universe by resorting to the extended thermodynamics of irreversible process. Moreover, Karami and Ghaffari [19] further extended the work of [18] and examine the validity of the generalized second-law in non-equilibrium (irreversible) thermodynamics in a non-flat FRW universe (with dynamical horizon) in which dark energy interacts with dark matter. In [20], Wang and Biao discussed the non-equilibrium thermodynamics of dark energy on cosmic apparent horizon. They argued that if the irreversible process is considered, the proper position for constructing thermodynamics will not be anymore the apparent horizon. The new position will be related with dark energy state equation and the irreversible process parameters.

In this paper, our aim is to extend the work done by Wang and Wen-Biao by introducing the power-law correction to the area-law which appear in dealing with the entanglement of quantum fields in and out the horizon. Considering the irreversible process, we find that the non-equilibrium thermodynamic laws can not be built on the original apparent horizon and develop a relationship which exhibits how the apparent horizon can be modified to keep the

non-equilibrium thermodynamic law in effect. The outline of the paper is as follows. Next Section is devoted to the non-equilibrium thermodynamics of dark energy on the power-law entropy corrected apparent horizon. Concluding remarks are given in Section III.

## II. NON-EQUILIBRIUM THERMODYNAMICS OF DARK ENERGY ON THE POWER-LAW ENTROPY CORRECTED APPARENT HORIZON

The metric of a homogeneous and isotropic universe in FRW model is

$$ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \quad (3)$$

where  $a(t)$  is the scale factor and  $k(= -1, 0, 1)$  is the constant curvature of their spatial section. By defining  $\tilde{r} = a(t)r$ , the location of the dynamical apparent horizon can be determined by setting  $f = 0$  in the relation

$$f = g^{ab}\tilde{r}_{,a}\tilde{r}_{,b} = 1 - \left( H^2 + \frac{k}{a^2} \right) \tilde{r}^2, \quad (4)$$

which is

$$\tilde{r} = \left( H^2 + \frac{k}{a^2} \right)^{-\frac{1}{2}}. \quad (5)$$

The observational data suggests that our universe is spatially flat with  $k = 0$ . In this case the apparent horizon is  $\tilde{r} = \frac{1}{H}$  which is equal to the Hubble horizon  $R_A$  i.e.,  $R_A = \frac{1}{H}$ .

We can define the surface gravity  $k$  and Hawking temperature respectively on the apparent horizon as [7]

$$k = -\frac{1}{2} \frac{\partial f}{\partial \tilde{r}} = \frac{\tilde{r}}{R_A^2}, \quad (6)$$

and

$$T = \frac{k}{2\pi} = \frac{\tilde{r}}{2\pi R_A^2}. \quad (7)$$

So the Hawking temperature at the apparent horizon has the form

$$T_A = T|_{\tilde{r}=R_A} = \frac{1}{2\pi R_A}. \quad (8)$$

Cai et al [21] showed that for an apparent horizon of an FRW universe, there exists Hawking radiations with temperature  $\frac{1}{2\pi R_A}$ . For the metric (3), the first Friedmann equation becomes

$$\left( \frac{\dot{a}}{a} \right)^2 = \frac{8}{3} \pi \rho, \quad (9)$$

where  $G = 1$  and  $\rho$  is the energy density of dark energy. The continuity equation for dark energy is

$$\dot{\rho} + 3H(1 + \omega)\rho = 0, \quad (10)$$

where  $\omega$  is the parameter of the equation of state of dark energy, and when  $\omega < -\frac{1}{3}$  it exhibits accelerating behavior of the universe. After defining  $\epsilon = \frac{3}{2}(1 + \omega)$ , for a definite equation of state, we obtain

$$a(t) = t^{\frac{1}{\epsilon}} \quad (0 < \epsilon < 1) \quad (-1 < \omega < -\frac{1}{3}), \quad (11)$$

and the apparent horizon can be written as  $R_A = \frac{1}{H} = \epsilon t$ .

The amount of energy flux acrossing the apparent horizon within the time interval  $dt$  is

$$-dE_A = 4\pi R_A^2 \rho(1 + \omega)dt = \epsilon dt. \quad (12)$$

Since the process of energy flux crossing the apparent horizon is irreversible, so an internal entropy production will be generated by this irreversibility. Thus, in the presence of interaction between dark energy and horizon with in, the time derivative of the non-equilibrium entropy is expressed as <sup>1</sup>

$$\dot{S} = \dot{S}_e + \dot{S}_i, \quad (13)$$

where  $\dot{S}_i$  shows the rate of change in internal entropy production of the universe while  $\dot{S}_e$  appears as heat flow between the universe and the horizon. If  $\dot{S}_e = 0$ , then this case corresponds to equilibrium thermodynamics i.e. the study of thermodynamics dealing with no transfer of energy or entropy across the physical boundary of the system. We mention here that irreversible (non-equilibrium) thermodynamics has been studied in the literature for various dynamical spacetimes in different gravitational theories [22].

Following the steps given in [20], we can have

$$\dot{S}_i = \oint_V \sigma dV, \quad (14)$$

$$\dot{S}_e = - \oint_{\Sigma} \vec{J}_s \cdot d\vec{\Sigma}, \quad (15)$$

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<sup>1</sup> In thermodynamics, the internal entropy is defined as the variation in entropy *inside* the thermal system, referring to irreversible processes, defined as  $dS = dS_e + dS_i$ , where  $dS_e$  is the entropy exchange with the surroundings or across the physical boundary of the system. Dividing by  $dt$  yields  $\frac{dS}{dt} = \frac{dS_e}{dt} + \frac{dS_i}{dt}$  or  $\dot{S} = \dot{S}_e + \dot{S}_i$ . In reversible thermodynamics, the entropy exchange term is zero and the total entropy is the internal entropy itself. Note that all natural processes are irreversible thus the physical universe as a whole is an irreversible thermal system.

where  $\sigma$  and  $\vec{J}_s$  are the internal entropy source production density and entropy flow density respectively. Again following [20], by considering the heat conduction between the universe and horizon,  $\sigma$  and  $\vec{J}_s$  can be expressed as

$$\sigma = \vec{J}_q \cdot \nabla \frac{1}{T} |_{\tilde{r}=R_A}, \quad (16)$$

$$\vec{J}_s = \frac{\vec{J}_q}{T_A}, \quad (17)$$

where  $\vec{J}_q$  is the heat current. After putting (17) in (15), we obtain

$$\dot{S}_e = -\frac{1}{T_A} \oint_{\Sigma} \vec{J}_q \cdot d\vec{\Sigma} = \frac{1}{T_A} J_q A, \quad (18)$$

where we have assumed that  $\vec{J}_q$  takes the same value at any point of the apparent horizon with surface  $A = 4\pi R_A^2$ . From the power-law entropy relation (1), we can write

$$\frac{d}{dt} \left[ \frac{A}{4} (1 - K_\alpha A^{(1-\frac{\alpha}{2})}) \right] = 2\pi R_A \dot{R}_A \left[ 1 - K_\alpha \left( \frac{4-\alpha}{2} \right) (4\pi R_A^2)^{1-\frac{\alpha}{2}} \right]. \quad (19)$$

Taking the relation

$$d(S) = d_e S, \quad (20)$$

we can have

$$J_q = \frac{\epsilon}{4\pi R_A^2} \left( 1 - K_\alpha \left( \frac{4-\alpha}{2} \right) (4\pi R_A^2)^{1-\frac{\alpha}{2}} \right), \quad \epsilon = \dot{R}_A. \quad (21)$$

According to the Fourier's law

$$\vec{J}_q = -\lambda \nabla T, \quad (22)$$

where  $\lambda$  is the thermal conductivity [20]. So there will be an energy flux  $\vec{J}_q$  if there is a  $\nabla T$  in a thermodynamical system. After substituting equations (21) and (22) in (16), we obtain

$$\sigma = \frac{\epsilon^2}{4\lambda R_A^2} \left( 1 - K_\alpha \left( \frac{4-\alpha}{2} \right) (4\pi R_A^2)^{1-\frac{\alpha}{2}} \right)^2. \quad (23)$$

So after inserting the value of  $\sigma$  in (14), we obtain

$$\dot{S}_i = \frac{\epsilon^2 \pi R_A}{3\lambda} \left( 1 - K_\alpha \left( \frac{4-\alpha}{2} \right) (4\pi R_A^2)^{1-\frac{\alpha}{2}} \right)^2. \quad (24)$$

Now from (19) and (24) the final expression of irreversible entropy can be written as

$$\begin{aligned} \dot{S} &= \dot{S}_e + \dot{S}_i \\ &= 2\pi R_A \epsilon \left( 1 - K_\alpha \left( \frac{4-\alpha}{2} \right) (4\pi R_A^2)^{1-\frac{\alpha}{2}} \right) \left( 1 + \frac{\epsilon}{6\lambda} - \frac{\epsilon}{6\lambda} K_\alpha \left( \frac{4-\alpha}{2} \right) (4\pi R_A^2)^{1-\frac{\alpha}{2}} \right) \end{aligned} \quad (25)$$

Therefore, the first law in irreversible thermodynamics holds if we define

$$dS = -\frac{dE}{\tilde{T}_A} = 2\pi\tilde{R}_A\epsilon dt, \quad (26)$$

where  $\tilde{T}_A = \frac{1}{2\pi\tilde{R}_A}$  and

$$\tilde{R}_A = R_A \left[ \left(1 - K_\alpha \left(\frac{4-\alpha}{2}\right) (4\pi R_A)^{1-\frac{\alpha}{2}}\right) \left(1 + \frac{\epsilon}{6\lambda} - \frac{\epsilon}{6\lambda} K_\alpha \left(\frac{4-\alpha}{2}\right) (4\pi R_A^2)^{1-\frac{\alpha}{2}}\right) \right], \quad (27)$$

which is also called power-law entropy corrected apparent horizon. To check consistency, if we put  $\alpha = 0$ , we obtain

$$\tilde{R}_A = R_A \left(1 + \frac{\epsilon}{6\lambda}\right), \quad (28)$$

which is the same as worked out in [20].

### III. CONCLUSION

In this paper, we considered the FRW universe and investigated non-equilibrium feature of it by taking the power-law correction to the horizon entropy. In an irreversible scenario, as energy goes outside the horizon, an internal entropy production term plays a significant role, which comes out to zero in the case of equilibrium. In [20], they showed that by considering the irreversible process, the non-equilibrium thermodynamic laws can not hold true on original apparent horizon and should be modified. They derived the expression for the modified apparent horizon which depends on the state parameter of dark energy  $\epsilon$  and a non-equilibrium factor  $\lambda$ . Here we extended their study by taking the power-law correction to entropy which appear in dealing with the entanglement of quantum fields in and out the horizon. The appearance of power-law correction terms to the horizon entropy is a fundamental prediction of quantum entanglement and must be considered in performing the thermodynamics study of horizon for the FRW universe. So from the power-law correction to entropy, we determined the entropy corrected form of the apparent horizon of the FRW universe.

Since the power-law entropy corrected apparent horizon is a cosmological horizon, hence it can be useful in variety of ways. For instance, in  $f(T)$  gravity, the thermodynamic properties of this horizon can be investigated following [23]. Also in fractional action cosmology, one can study the generalized second law of thermodynamics at the the power-law entropy corrected apparent horizon following [24]. Similarly one can investigate thermal properties

of interacting holographic and new-agegraphic dark energy models with corrected-apparent horizon [25].

### *Acknowledgment*

We would like to thank the referees for giving useful comments on our work.

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